

Local anisotropy of space in a frame of reference co-moving with the Earth

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Abstract

We consider, in the framework of General Relativity, the linear approximation of the gravitational field of the Earth taking into account its mass, its quadrupole moment, its shape and its diurnal rotation.

We conclude that in the frame of reference co-moving with the Earth the local anisotropy of the space is of the order of $10^{-12} - 10^{-13}$ and could be observed.

1 Introduction

Three different types of experiments have been performed to test the isotropy of space. Although this concept has always a geometrical connotation what is really meant depends crucially on the type of experiments being considered.

The first type of experiments are those of the type pioneered by Michelson and Morley [1] and developed by Miller [2], Joos [3] and many others using optical interferometers. Jaseja and al. [4] introduced a new setup using a stabilized laser as a standard of frequency and a Fabry-Perot as a standard of length, a technique that was used again by Brillet and Hall [5] who claim to have attained the best sensitivity.

Historically the purpose of these experiments was to measure the absolute velocity of the Earth, i.e. the velocity with respect to that particular frame

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of reference for which the light propagation, according to Newtonian Physics, was isotropic. In these experiments $\alpha(t) = \Delta c/c$, where Δc is the difference of the round-trip speed of light along two perpendicular directions, say south and east, on a horizontal plane, is measured as a function of time t . These experiments¹ have consistently yield results for $\max .\alpha(t)$ ranging from 10^{-9} to 10^{-13} .

The second type of experiments was pioneered by Kennedy and Thorndike [6] and is a variant of the Michelson-Morley one in the sense that light travels along a single direction fixed with respect to the Earth and one attempts to measure $\beta(t) = (c(t) - c(t_0))/c(t_0)$, where $c(t)$ is the round-trip speed of light along the fixed direction.

The greatest value for $\alpha(t)$ obtained from the Michelson-Morley experiment is already 10 times smaller than the expected maximum Newtonian value 10^{-8} and therefore these experiments are considered to be good tests of Special Relativity. In fact, as Jaseja and al. have pointed out is not so obvious to decide what these experiments test. More precisely we ask here whether they should be analyzed in the framework of Special Relativity, ignoring or not the centrifugal force due to the Earth's rotation?, or should this analysis have to take into account the Coriolis field and the Newtonian gravitational field in the framework of General Relativity?

In the first case the Michelson-Morley experiment can be considered to be a test of isotropy of space in a cosmic global sense. In the second case the same experiment has to be considered as a test of the theory of light propagation in the presence of local inertial and gravitational fields. On the contrary the Kennedy-Thorndike experiment can only be considered as a test of the isotropy of space in the cosmic global sense.

The distinction between the two points of view would be of paramount importance if a future experiment of one of these types yields a significant non-null result at some precision. The first point of view would mean the failure of Special Relativity while the second one does not mean that. It means that Special Relativity as a global model of space-time is not applicable here and that gravitational fields interfere with light already at a local level.

The third type of experiments was pioneered by Drever and Hughes [7].

¹From a careful reading of [5] it follows that the upper limit is 10^{-13} if one assumes that the anisotropy could have a local origin and 10^{-15} if one assumes that the anisotropy is necessarily a cosmic one.

They test the variation of an atomic or nuclear level splitting due to a fixed magnetic field as the orientation of this field with respect to the stars changes due to the Earth's rotation and motion around the Sun. These experiments yield a limit to this type of anisotropy of the order of 10^{-20} , a limit that Lamoreaux and al. [8] pushed to 10^{-22} . Because the setup of these type of experiments is fixed with respect to the Earth they can be compared with the Kennedy-Thorndike type, although the physics which is involved is quite different. But they cannot be compared to the Michelson-Morley type because this one can discriminate between three cases: i) $\alpha(t) = 0$, ii) $\alpha = \text{constant} \neq 0$ and iii) $\alpha(t)$ a function of time.

In this paper we analyze the local anisotropy of space in the neighborhood of the Earth's surface. The first section is based on the theory of Principal transformations. The analysis of the anisotropy of space is based on the exterior field only. The value of α predicted depends on the latitude of the site where the experiment is performed and three parameters close to the mean radius of the Earth.

The analysis of the second section is global but model dependent. We have considered a model of the Earth with the observed oblateness but assumed its density to be constant. This analysis depends also on some heuristics that can be justified by the theory of Principal transformations.

The third section contains a remainder of our interpretation of the anisotropy of space and a numerical comparison of the results derived from the approaches of the two preceding sections.

2 The gravitational field of the Earth: A local approach.

As an approximate model of the gravitational field of the Earth in the vacuum neighborhood of its surface we consider the following line-element ²:

$$ds^2 = -(1 - 2U_G)dt^2 + (1 + 2U_G)(dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)) \quad (1)$$

where U_G is:

$$U_G = \frac{M}{r} \left(1 + \frac{1}{2} \frac{J_2 R^2 (1 - 3 \cos^2 \theta)}{r^2} \right) \quad (2)$$

²We shall be using units such that $c = 1$ and $G = 1$.

M being the mass of the Earth, J_2 being the coefficient of its gravitational quadrupole moment and R the equatorial radius of the ellipsoidal surface. The “polar” coordinates (r, θ, φ) have been chosen for convenience and are related to a system of harmonic “Cartesian” ones (x^i) by the usual formulas:

$$x^1 = r \sin \theta \cos \varphi, \quad x^2 = r \sin \theta \sin \varphi, \quad x^3 = r \cos \theta \quad (3)$$

Considering only those terms which are linear with respect to M and/or J_2 the line-element 1 is a solution of Einstein’s vacuum equations which actually reduce to the Newtonian equation:

$$\Delta U_G = 0 \quad (4)$$

To take into account the Earth’s rotation Ω we select a new frame of reference related to the preceding one by the transformation:

$$\varphi \rightarrow \varphi + \Omega t \quad (5)$$

The line-element 1 becomes:

$$ds^2 = -(1 - 2(U_G + U_\Omega))dt^2 + 2A_\varphi dt d\varphi + (1 + 2U_G)(dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)) \quad (6)$$

where:

$$U_\Omega = \frac{1}{2}\Omega^2 r^2 \sin^2 \theta, \quad A_\varphi = \Omega r^2 \sin^2 \theta \quad (7)$$

Introducing new “cartesian coordinates” (x^i) connected with the new “polar” coordinates by the same formula 5 the line-element 6 becomes:

$$ds^2 = -(1 - 2U)dt^2 + 2A_i dx^i dt + (1 + 2U_G)\delta_{ij} dx^i dx^j \quad (8)$$

where:

$$U = U_G + U_\Omega, \quad (9)$$

and the non zero components of A_i are:

$$A_1 = -\Omega y, \quad A_2 = \Omega x. \quad (10)$$

From now on we shall neglect any terms proportional to powers of Ω greater than 2.

Let τ be the proper time along a time-like geodesic then, neglecting cubic and higher order terms in the velocities, the free fall of a test particle is described by the following equations:

$$\frac{d^2 x^k}{d\tau^2} + \hat{\Gamma}_{ij}^k \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} = \left(1 + \delta_{ij} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \right) \Lambda^k + 2\Omega_j^k \frac{dx^j}{d\tau} \quad (11)$$

where the Newtonian and Coriolis field are respectively:

$$\Lambda^i = \delta^{ij} \partial_j (U_G + U_\Omega), \quad \Omega_j^i = \frac{1}{2} \delta^{is} (\partial_s A_j - \partial_j A_s) \quad (12)$$

and:

$$\hat{\Gamma}_{ij}^k = \delta_j^k \partial_i U_G + \delta_i^k \partial_j U_G - \delta_{ij} \delta^{ks} \partial_s U_G - (\Omega_i^k \Omega_{jl} + \Omega_j^k \Omega_{il}) x^l \quad (13)$$

Notice that, whether Ω is taken into account or not, Eqs. 11 are meaningless without clarifying the meaning of the coordinates being used. Notice also that the newly defined “cartesian” coordinates x^i are no longer harmonic coordinates.

The symbols $\hat{\Gamma}_{ij}^k$ in 11 are the Christoffel symbols corresponding to the Riemannian metric:

$$d\hat{s}^2 = ((1 + 2U_G)\delta_{ij} + \Omega_{ik}\Omega_{jl}x^kx^l)dx^i dx^j \quad (14)$$

which is the quotient of the space-time metric 6 by the Killing congruence of the frame of reference.

This object is usually interpreted as defining the “geometry” of the space for the frame of reference being considered. This interpretation is correct or not depending on the meaning that one attaches to this word. It is certainly correct in a technical, current and accepted sense meaning a well established branch of mathematics. It is not correct if by “geometry of space” we mean that particular Riemannian geometry which allows the description of rigid objects which can be displaced, or compared when they occupy different positions in space. This point has been strongly emphasized by Poincaré with profound insight [9], and requires the geometry of space to have constant curvature [10], the simplest case being of course the case in which this geometry is flat.

If the object 14 is not the metric of space, what is it?. Before reminding a new interpretation which was first proposed in [11] we are going to state and solve the problem of constructing the Principal transformation of the

Riemannian metric 14, this being a concept introduced by one of us (Ll.B) in [12].

Using polar coordinates we have:

$$d\hat{s}^2 = (1 + 2U_G)(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta (1 + 2U_\Omega) d\varphi^2) \quad (15)$$

The principal directions, i.e. the eigen-vectors of \hat{R}_{ij} , the Ricci tensor of this metric with respect to \hat{g}_{ij} are to the required approximation proportional to:

$$n_{1i} = (1, J_2 f, 0), \quad n_{2i} = (-J_2 f, 1, 0), \quad n_{3i} = (0, 0, 1) \quad (16)$$

where we have used the euclidean orthonormal co-basis $(dr, r d\theta, r \sin \theta d\varphi)$ and where:

$$f = -4 \frac{R^2}{r^2} \left(1 - q \frac{r^5}{R^5} \right) \sin \theta \cos \theta \quad \text{with} \quad q = \frac{1}{4} \frac{\Omega^2 R^3}{M J_2} \quad (17)$$

The vector n_1 is up to the required approximation, as a short calculation shows, collinear with the gradient of U pointing outwards. The vector n_2 is therefore that vector lying in the meridian of the point being considered which is orthogonal to the vertical pointing south, and n_3 is the the vector lying in the corresponding parallel and pointing east.

Using the preceding vectors as a basis of an orthogonal decomposition the line-element 15 can be written:

$$d\hat{s}^2 = (\hat{\theta}^1)^2 + (\hat{\theta}^2)^2 + (\hat{\theta}^3)^2 \quad (18)$$

where:

$$\hat{\theta}^1 = (1 + U_G)(dr + J_2 f r d\theta), \quad \hat{\theta}^2 = (1 + U_G)(-J_2 f dr + r d\theta), \quad \hat{\theta}^3 = (1 + U) r \sin \theta d\varphi \quad (19)$$

By definition, in this particular case, a Principal transform of 18 is a metric which can be written as:

$$d\bar{s}^2 = c_1^2 (\hat{\theta}^1)^2 + c_2^2 (\hat{\theta}^2)^2 + c_3^2 (\hat{\theta}^3)^2 \quad (20)$$

the three functions $c_i(x^j)$ being chosen such that:

i) the metric 20 is euclidean, i.e. such that:

$$\bar{R}_{jkl}^i = 0 \quad (21)$$

ii) the following “quo-harmonicity condition” is satisfied:

$$(\bar{\Gamma}_{jk}^i - \hat{\Gamma}_{jk}^i) \hat{g}^{jk} = 0 \quad (22)$$

where $\bar{\Gamma}_{jk}^i$ are the Christoffel symbols of the line-element 20, and \hat{g}^{jk} is the inverse matrix of \hat{g}_{jk} .

Notice that both groups of conditions are tensor equations and therefore the solutions $c_i(x^j)$ are scalar functions independent of the system of coordinates being used, as long as this system is adapted to the Killing congruence defining the static frame of reference.

Integrating 21 and 22 we have obtained to the required approximation:

$$\begin{aligned} c_1 &= 1 - \frac{M}{r} \left(1 + \frac{1}{5} \frac{R_1^2}{r^2} \right) + \frac{MJ_2 R^2}{r^3} \left(6 + \frac{6}{5} \frac{R^2}{r^2} - \left(5 + \frac{9}{5} \frac{R^2}{r^2} \right) \sin^2 \theta \right) \\ &\quad + \Omega^2 r^2 \left(-\frac{27}{5} + \frac{3}{10} \frac{R^2}{r^2} + \left(\frac{15}{2} - \frac{3}{10} \frac{R^2}{r^2} \right) \sin^2 \theta \right) \\ c_2 &= 1 - \frac{M}{2r} \left(3 - \frac{1}{5} \frac{R_2^2}{r^2} \right) + \frac{MJ_2 R^2}{r^3} \left(1 - \frac{3}{5} \frac{R^2}{r^2} - \left(\frac{19}{4} - \frac{21}{20} \frac{R^2}{r^2} \right) \sin^2 \theta \right) \\ &\quad + \Omega^2 r^2 \left(-\frac{19}{5} + \left(\frac{13}{2} + \frac{3}{10} \frac{R^2}{r^2} \right) \sin^2 \theta \right) \\ c_3 &= 1 - \frac{M}{2r} \left(3 - \frac{1}{5} \frac{R_2^2}{r^2} \right) + \frac{MJ_2 R^2}{r^3} \left(1 - \frac{3}{5} \frac{R^2}{r^2} - \left(\frac{9}{4} + \frac{3}{4} \frac{R^2}{r^2} \right) \sin^2 \theta \right) \\ &\quad + \Omega^2 r^2 \left(-\frac{19}{5} + 4 \sin^2 \theta \right) \end{aligned}$$

where R_1 and R_2 are in this approach two model-dependent free parameters. They should be determined by matching the preceding results with the corresponding ones in the interior of the Earth. This would require a global \mathcal{C}^2 model including the interior gravitational field and therefore also a \mathcal{C}^0 behavior of the density of the Earth. To deal with a discontinuous density requires to adopt a different approach. This is done in the next section.

3 The gravitational field of the Earth: A global approach.

To begin with we consider in this section a model of the Earth with constant density taking into account its oblateness. More precisely we shall assume that the surface of the Earth Σ is $r^2 = a^2(1 + \lambda^2 \sin^2 \theta)$, where a is the polar radius, and $\lambda = \sqrt{R^2 - a^2}/R$ the eccentricity, R as before is the equatorial radius.

An elementary integration of the corresponding Poisson equation yields up to order J_2 the Newtonian potential:

$$U_G = \begin{cases} \frac{M}{r} \left(1 + \frac{J_2 R^2}{2 r^2} (1 - 3 \cos^2 \theta) \right) & \text{outside} \\ \frac{M}{R} \left(\frac{3}{2} - \frac{1}{2} \frac{r^2}{R^2} + \frac{J_2}{2} \left(\frac{5}{2} - \frac{3}{2} \frac{r^2}{R^2} - 3 \frac{r^2}{R^2} \cos^2 \theta \right) \right) & \text{inside} \end{cases} \quad (23)$$

which is of class \mathcal{C}^1 across Σ , where $J_2 = \lambda^2/5$.

Taking into account also the diurnal rotation of the Earth, the gravitational field in a comoving frame of reference is then described by the line element 8, where now U_G is given by 23 instead of 2 and correspondingly we have to make the same substitution in 15.

On the surface of the Earth Σ the Ricci tensor of the metric 15 is discontinuous as it is the density and therefore the principal directions derived from the exterior field do not coincide with those derived from the interior one. This prevents the existence for this metric of a global principal transformation continuous across Σ .

When a principal transformation exists then from 22 it follows that cartesian coordinates of 20, i. e. those with $\bar{\Gamma}_{jk}^i = 0$ are quo-harmonic coordinates of 15, i. e. they are such that $\hat{\Gamma}_{jk}^i \hat{g}^{ij} = 0$. The approach will then be heuristically justified by this result. We shall obtain a global system of quo-harmonic coordinates, unique up to euclidean transformations, and we shall consider the anisotropy of the space to be the anisotropy of 15 with respect to the euclidean metric which in this system of coordinates has components $\bar{g}_{ij} = \delta_{ij}$.³

³This point of view was already used in [14] where harmonic coordinates of the space-time were considered instead of the quo-harmonic coordinates of the space. It could be considered also in connection with rotating Fermi coordinates based on the world-line of

Under an infinitesimal coordinate transformation

$$x^i = z^i + \zeta^i(z^j)$$

where ζ_i are of the order appropriate to the approximation that we consider, the Riemannian metric 14 becomes:

$$\hat{g}'_{ij} = \hat{g}_{ij}(x^k(z^l)) + \frac{\partial \zeta_j}{\partial z^i} + \frac{\partial \zeta_i}{\partial z^j}.$$

The new coordinates z_i will be quo-harmonic if: [13]

$$\Delta \zeta_k(z^i) = \frac{\partial U}{\partial z^k}. \quad (24)$$

We can solve this equation splitting the functions ζ_k in their gravitational and rotational part $\zeta_k = \zeta_k^G + \zeta_k^\Omega$. Then

$$\Delta \zeta_k^G = \frac{\partial U_G}{\partial z^k}, \quad (25)$$

$$\Delta \zeta_k^\Omega = -2\Omega_{kj}\Omega^j_l z^l. \quad (26)$$

The solution for ζ_k^G can found by integrating the Poisson equation 25 because the “density” $\partial U_G/\partial z^k$ is known. Requiring the gradient of ζ_k^G to be zero at infinity. In the new polar coordinates, i.e. coordinates (r, θ, φ) such that:

$$z^1 = r \sin \theta \cos \varphi, \quad z^2 = r \sin \theta \sin \varphi, \quad z^3 = r \cos \theta \quad (27)$$

we obtain:

$$\zeta_r^G = \begin{cases} M \left(\frac{1}{2} - \frac{1}{10} \frac{R^2}{r^2} + J_2 \frac{R^2}{r^2} \left(\frac{3}{14} \frac{R^2}{r^2} + \frac{1}{4} \left(1 - \frac{9}{7} \frac{R^2}{r^2} \right) \sin^2 \theta \right) \right) & \text{outside} \\ M \frac{r}{R} \left(\frac{1}{2} - \frac{1}{10} \frac{r^2}{R^2} + J_2 \left(\frac{3}{4} - \frac{15}{28} \frac{r^2}{R^2} - \frac{1}{2} \left(1 - \frac{6}{7} \frac{r^2}{R^2} \right) \sin^2 \theta \right) \right) & \text{inside} \end{cases} \quad (28)$$

the center of the Earth. These two coordinate dependent models are heuristic as the one presented in this section. The stronger legitimacy of the latter comes from being akin with the method of the first section.

$$\zeta_\theta^G = \begin{cases} -\frac{M}{2} J_2 \frac{R^2}{r} \sin \theta \cos \theta \left(1 - \frac{3}{7} \frac{R^2}{r^2}\right) & \text{outside} \\ -\frac{M}{2} J_2 \frac{r^2}{R} \sin \theta \cos \theta \left(1 - \frac{3}{7} \frac{r^2}{R^2}\right) & \text{inside} \end{cases} \quad (29)$$

The rotational part can be obtained very easily from an inspection of the explicit expression of 26. Demanding the appropriate symmetry and the correct limit on the axis of symmetry we obtain:

$$\zeta_r^\Omega = \frac{1}{4} \Omega^2 r^3 \sin^4 \theta \quad \zeta_\theta^\Omega = \frac{1}{4} \Omega^2 r^4 \sin^3 \theta \cos \theta \quad (30)$$

Writing the metric 14 as $\hat{g}_{ij} = \delta_{ij} + \hat{h}_{ij}$ and using polar coordinates we obtain for \hat{h}_{ij} :

$$\begin{aligned} \hat{h}_{rr} &= 2 \frac{M}{r} \left(1 + \frac{1}{5} \frac{R^2}{r^2} + J_2 \frac{R^2}{r^2} \left(-1 - \frac{6}{7} \frac{R^2}{r^2} + \left(1 + \frac{9}{7} \frac{R^2}{r^2} \right) \sin^2 \theta \right) \right) + \\ &\quad \frac{3}{2} \Omega^2 r^2 \sin^4 \theta, \\ \hat{h}_{\theta\theta} &= Mr \left(3 - \frac{1}{5} \frac{R^2}{r^2} - J_2 \frac{R^2}{r^2} \left(-3 + \frac{6}{7} \frac{R^2}{r^2} + \left(\frac{11}{2} - \frac{3}{2} \frac{R^2}{r^2} \right) \sin^2 \theta \right) \right) + \\ &\quad \frac{3}{2} \Omega^2 r^4 \sin^2 \theta \cos^2 \theta, \\ \hat{h}_{\phi\phi} &= Mr \sin^2 \theta \left(3 - \frac{1}{5} \frac{R^2}{r^2} + J_2 \frac{R^2}{r^2} \left(-3 + \frac{6}{7} \frac{R^2}{r^2} + \left(\frac{9}{2} - \frac{15}{14} \frac{R^2}{r^2} \right) \sin^2 \theta \right) \right) + \\ &\quad \frac{3}{2} \Omega^2 r^4 \sin^4 \theta, \\ \hat{h}_{r\theta} &= \sin \theta \cos \theta \left(2M J_2 \frac{R^2}{r^2} \left(1 - \frac{6}{7} \frac{R^2}{r^2} \right) + \frac{3}{2} \Omega^2 r^3 \sin^2 \theta \right). \end{aligned} \quad (31)$$

Making more precise what we said before we define the eigen-values and the principal directions of the anisotropy of the metric 15 as the scalars c_a and the vectors n_a^i solutions of the algebraic equations:

$$(\hat{g}_{ij} - c_a \delta_{ij}) n_a^j = 0.$$

with \hat{h}_{ij} given above by 31. To the required approximation the scalars c_a are:

$$c_1 = 1 + \hat{h}_{rr}, \quad c_2 = 1 + \frac{\hat{h}_{\theta\theta}}{r^2}, \quad c_3 = 1 + \frac{\hat{h}_{\phi\phi}}{r^2 \sin^2 \theta} \quad (32)$$

and the components of the principal directions in the euclidean orthonormal co-basis $(dr, r d\theta, r \sin \theta d\varphi)$ are proportional to:

$$n_{1i} = (1, f(r, \theta), 0), \quad n_{2i} = (-f(r, \theta), 1, 0), \quad n_{3i} = (0, 0, 1) \quad (33)$$

where:

$$f(r, \theta) = \frac{-\hat{h}_{r\theta}}{M(1 - 3R^2/(5r^2))}.$$

Conclusion

In section 2 we exhibited the anisotropy of the riemannian metric 15 with respect to an intrinsically associated euclidean metric. This association depends on three model dependent parameters R_1, R_2 and R . If the quadrupole moment were zero the three parameters would coincide and assuming a model of the Earth with constant density their value would be its radius. Therefore we can safely guess that the three parameters will be close to each other and of the order of the mean radius of the Earth. It is worthwhile to notice that on any horizontal plane, i.e orthogonal to the gradient of U the anisotropy depends only on R . And in particular if $r = R \approx R_1 \approx R_2$ we get:

$$\alpha = c_2 - c_3 = -\frac{1}{5} \left(\frac{11MJ_2}{R} - 14\Omega^2 R^2 \right) \sin^2 \theta \quad (34)$$

Assuming the following numerical values: $M = 0.00444$ $R = 6378164$ and $\Omega = 2.434 \times 10^{-13}$, the three of them measured in meters, and $J_2 = 0.0010826$ dimensionless, at a latitude of 45° we obtain:

$$\alpha = c_2 - c_3 = 2.5 \times 10^{-12} \quad (35)$$

In section 3 we constructed a globally \mathcal{C}^1 model of the gravitational field of the Earth assuming it to be of constant density and having an ellipsoidal shape.⁴ Then we obtained a global system of quo-harmonic coordinates of

⁴This model of the Earth would yield a value of $\lambda^2 = 0.00541$. while the observed value is $\lambda^2 = 0.00674$

the space. It follows from that that we can associate with the global field of the Earth an euclidean metric with components $\bar{g}_{ij} = \delta_{ij}$ in this unique system of quo-harmonic coordinates. This is again a model dependent but intrinsic association.

With this model the anisotropy in the horizontal plane, for $r = R$ is:

$$\alpha = c_2 - c_3 = \left(-\frac{4}{7} \frac{M J_2}{R} + \frac{3}{2} \Omega^2 R^2 \right) \sin^2 \theta \quad (36)$$

and with J_2 , M , R , Ω and θ as before, this formula yields:

$$\alpha = c_2 - c_3 = 6.2 \times 10^{-13} \quad (37)$$

The two results in 35 and 37 differ, although by less of a factor of 10. The safe conclusion that we can draw from our precedent analysis is that it predicts an anisotropy on the horizontal plane in the range $10^{-12} - 10^{-13}$.⁵ It is however important to remind that the theory of Principal transformations tell us that for a model of the gravitational field of the Earth of class \mathcal{C}^2 the two methods would yield the same result.

The important question is now the following: how could this anisotropy show up?. Because the anisotropy is not due to any cosmic effect but is due to the gravitational field of the Earth and to its diurnal rotation and because it is relative to a co-moving frame of reference, no experiment of the Kennedy-Thorndike or Hughes-Drever type could reveal it. If instead, as it was first proposed in [11], [14] and [15], we renounce to interpret the metric 15 with components \hat{g}_{ij} as the geometry of space, a role which belongs to the euclidean metric \bar{g}_{ij} , and we interpret the eigen-values of the first of this objects with respect to the second as the velocities of light along the principal directions of an anisotropic propagation, then the experiments of the Michelson-Morley type should reveal such anisotropy.

As a matter of fact Brillet and Hall, the authors of the most recent repetition of this experiment obtained a raw result of the order of 2.1×10^{-13} at the latitude of $+40^\circ$. If confirmed our model could provide an explanation for this result. In our opinion is an urgent task that somebody repeats this experiment using their setup or any other more appropriate variant.

⁵The harmonic coordinate method and the Fermi coordinate one yield values in the same range.

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